Thermodynamics of the heat currents in the longitudinal spin Seebeck and spin Peltier effects

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Abstract

We employ the non-equilibrium thermodynamics of currents and forces to describe the heat transport caused by a spin current in a Pt/YIG bilayer. By starting from the constitutive equations of the magnetization currents in both Pt and YIG, we derive the magnetization potentials and currents. We apply the theory to the spin Peltier experiments in which a spin current, generated by the spin Hall effect in Pt, is injected into YIG. We find that efficient injection is obtained when: i) the thickness of each layer is larger than its diffusion length: $t_{Pt} > l_{Pt}$ and $t_{YIG} > l_{YIG}$ and ii) the ratio $(l_{Pt}/\tau_{Pt})/(l_{YIG}/\tau_{YIG})$ is small, where τ_i is the time constant of the intrinsic damping $(i = Pt, YIG)$. We finally derive the temperature profile in adiabatic conditions. The scale of the effect is given by the parameter ΔT_{SH} which is proportional to the electric current in Pt. Using known parameters for Pt and YIG we estimate $\Delta T_{SH}/j_e = 4 \cdot 10^{-13}$ K A^{-1}m^2. This value is of the same order of magnitude of the spin Peltier experiments.

I. INTRODUCTION

The longitudinal spin Seebeck effect (LSSE), measured in ferrimagnetic insulators like yttrium iron garnet (YIG), is the consequence of the transport properties of out-of-equilibrium magnons which are at the same time responsible for heat and spin currents[1]. Most of the recent experiments and theories have dealt with the spin potential generated by the spin Seebeck effect in a YIG layer. This potential is able to inject a spin current into an adjacent Pt layer, where it is revealed by means of the inverse spin Hall effect. However, in the same way as in the thermoelectric effects, there should exist the reciprocal of the spin Seebeck effect which can be called the spin Peltier effect. In this second case a magnetization current injected into YIG will cause a heat current. This spin Peltier effect[2] has been recently observed. In the spin Peltier experiments the spin Hall effect of Pt is used as spin current injection and one observes the thermal effects caused by the magnetization current forced inside YIG, so the spin Peltier experiment becomes perfectly reciprocal to the spin Seebeck one.

On the basis of non-equilibrium thermodynamics[3], it is possible to derive the constitutive equations for the magnetization and heat transport in the insulating ferrimagnet[4]. With respect to other approaches,[5–7] we aim to describe the magnetization transport process by means of the effective force associated with the magnetization current, independently of the specific carriers (electrons or magnons). In the present work we apply this theory, together with the spin Hall effect theory, to describe the heat transport caused by spin Peltier effect. The spin Peltier heat current term is $\epsilon_M T j_M$, where ϵ_M is the spin Seebeck coefficient, T the absolute temperature and j_M the magnetization current. By starting from the constitutive equations of the magnetization currents in both Pt and YIG we derive the magnetization potentials. In particular we employ here adiabatic conditions at the YIG layer. In this way we are able to obtain the analytical expressions for the profiles of both magnetization current and temperature.

The results show that, since the YIG magnetization current is concentrated within the magnetic diffusion length of the YIG, the thermal effects are only active within l_{YIG}, which is estimated to be in the micron range. The temperature profile under adiabatic conditions depends on both the value of the spin Seebeck coefficient ϵ_M and by the ratio between the thermal conductivity due to magnons $\kappa_M = \epsilon_M^2 \sigma_M T$ and the total thermal conductivity

$\kappa_M + \kappa$, where in κ one includes all other heat transport mechanisms. By using the values for YIG derived in a previous analysis $\epsilon_M \sim 10^{-2}$ TK^{-14}, we derive $\kappa_M \sim 10^{-2}$ W $\mathrm{K}^{-1}\mathrm{m}^{-1}$. We also obtain the typical scale of amplitude of the spin Peltier effect by means of the parameter ΔT_{SH} which is proportional to the electric current in Pt. For a Pt injection into YIG we find $\Delta T_{SH}/j_e = 4 \cdot 10^{-13}$ K $\mathrm{A}^{-1}\mathrm{m}^2$. This value is of the same order of magnitude of the Spin Peltier experiments and more specifically we compare the values to the experiments of Refs.[2,8].

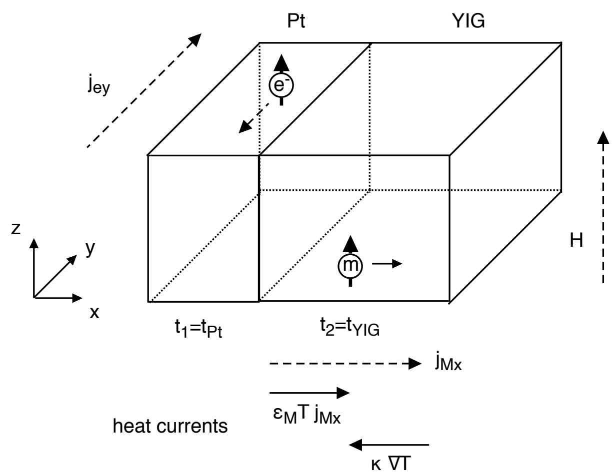

FIG. 1. Geometry of the spin Peltier effect. The electric current in Pt is along y, the magnetization current and the thermal effects are along x, the direction of the magnetic moments is along z.

II. THEORY

From the thermodynamics viewpoint, the transport of magnetic moment (magnetization current) can be analyzed by making an analogy (or an extension) of the thermoelectric effects by considering the magnetic moment at the place of (or in addition to) the charge[9]. One of the key points is that the magnetization current density j_M is not continuous and therefore one needs to provide a continuity equation for the magnetization. Starting from the thermodynamic approach of Johnson and Silsbee[3] and using scalar magnetization one defines

$$\frac{\partial M}{\partial t} + \nabla \cdot \boldsymbol{j}_M = \frac{H - H_{eq}}{\tau_M} \tag{1}$$

where M is the magnetization (the volume density of magnetic moment, measured in Am^{-1}), $\boldsymbol{j}_M$ is the magnetization current density, H is the magnetic field, $H_{eq}(M)$ the magnetic equation of state at equilibrium and τ_M is a relaxation time associated with the damping process. The equation means that every time the field H is not equal to the equilibrium value H_{eq}, either the local magnetization changes in time or a magnetization current is established. By developing the non equilibrium thermodynamics of fluxes and forces one defines the thermodynamic driving force of the magnetization current as the gradient of the effective potential ∇H^* with $H^* = H - H_{eq}$. This definition permits to derive the profiles of the magnetization current along different media and is independent of the specific carriers of the magnetization current (either electrons or magnons).

In the paper we apply the theory to the geometry of a bilayer composed by Pt and YIG. The Pt is the active layer where the magnetization current is generated by means of the spin Hall effect. The YIG is the passive layer in which the magnetization current is injected. Because of the spin Peltier effects of YIG, the injection of the magnetization current also forces a heat current inside the YIG. Here we are interested to the temperature profile which is generated in stationary and adiabatic conditions.

A. Platinum

Platinum is a non magnetic metal with a strong spin Hall effect. Both the electric and the magnetization currents are carried by electrons. We need to specify the relation between the currents and the forces in two dimensions. In presence of the spin Hall effect we have two equations relating the electric current j_{ey} along y and the magnetization current j_{Mx} along x to the gradients of the magnetic and electric potentials. By adapting the theory of the spin Hall effect to the symbols used here, we have the electric equation

$$j_{ey}/\sigma_0 = -\nabla_y V_e + \theta_{SH}\mu_0(\mu_B/e)\nabla_x H^* \tag{2}$$

with $\nabla_x = \partial/\partial x$ and $\nabla_y = \partial/\partial y$ and the magnetic equation

$$(e/\mu_B) j_{Mx}/\sigma_0 = \theta_{SH}\nabla_y V_e + \mu_0(\mu_B/e)\nabla_x H^* \tag{3}$$

where θ_{SH} is the spin Hall angle which is negative for the spin-orbit interaction of conduction electrons in metals, σ_0 is the conductivity for electron transport, e is the elementary charge,

μ_B is the Bohr magneton and μ_0 is the permeability of vacuum. By eliminating $\nabla_y V_e$ we have

$$j_{Mx} = j_{SH} + \sigma'_M \mu_0 \nabla_x H^* \tag{4}$$

where

$$j_{SH} = -(\mu_B/e)\theta_{SH} j_{ey} \tag{5}$$

is the magnetization current due to the spin Hall effect and

$$\sigma'_M = (\mu_B/e)^2 \sigma_0 (1 + \theta_{SH}^2) \tag{6}$$

is an effective magnetic conductivity inside Pt. The magnetic equation must be solved together with the continuity equation under stationary conditions

$$\nabla_x j_{Mx} = \frac{H^*}{\tau_M} \tag{7}$$

to give the diffusion equation for the magnetic potential inside Pt

$$l_M^2 \nabla_x^2 H^* = H^* \tag{8}$$

where

$$l_M = \sqrt{\mu_0 \sigma'_M \tau_M} \tag{9}$$

is the diffusion length inside Pt. The solution is obtained by setting boundary conditions with the surrounding media.

B. YIG

YIG is an electrical insulator with ferrimagnetic order. The magnetization current is carried by non equilibrium magnons which are also responsible, in part, for the conduction of heat. In YIG we need to relate the magnetization current and the heat current to the associated forces. Here we limit to currents and forces in one dimension (x). The equations are

$$j_M = \sigma_M \, \mu_0 \nabla_x H^* - \epsilon_M \sigma_M \, \nabla_x T \tag{10}$$

and

$$j_q = \epsilon_M \sigma_M T \mu_0 \nabla_x H^* - (\kappa + \kappa_M) \nabla_x T \tag{11}$$

where σ_M is the spin conductivity, ϵ_M is the spin Seebeck coefficient, j_q is the heat current density, κ is the thermal conductivity with $j_M = 0$ and

$$\kappa_M = \epsilon_M^2 \sigma_M T, \tag{12}$$

is the contribution of magnons to the thermal conductivity. The heat diffusion equations in stationary conditions is

$$\nabla_x j_q = \mu_0 \nabla_x H^* j_M + \frac{\mu_0 \left(H^*\right)^2}{\tau_M} \tag{13}$$

where the terms at the right hand side are due to the energy dissipation of the magnetization current and to the local damping, respectively. Both terms are essentially quadratic in the force and the potential. Therefore if we assume small currents and forces we are allow to neglige them in a first approximation. In this case we obtain the condition $\nabla_x j_q = 0$ which, in one dimension, corresponds to a constant heat flux traversing the YIG layer. As the second equation can also be written using Eq.(10) as

$$j_q = \epsilon_M T j_M - \kappa \nabla_x T \tag{14}$$

we choose here to study the adiabatic condition in which $j_q = 0$ in which the temperature profile $T(x)$ inside YIG will counterbalance the spin Peltier term $\epsilon_M T j_M$ giving no net heat flow. The profile will be stable if the thermal bath at the two ends of the YIG layer are let free to adapt at the temperatures of the two ends. From Eq.(11) with $j_q = 0$ we have

$$\nabla_x T = \frac{1}{\epsilon_M} \frac{\kappa_M}{\kappa + \kappa_M} \mu_0 \nabla_x H^* \tag{15}$$

and from Eq.(10) with the continuity equation for the magnetization current (Eq.7), we have the diffusion equation as Eq.(8) where the diffusion length in adiabatic conditions l_M is related to the diffusion length $l_{M0} = \sqrt{\mu_0 \sigma_M \tau_M}$ by

$$l_M = l_{M0} \sqrt{1 - \frac{\kappa_M}{\kappa + \kappa_M}}. \tag{16}$$

III. RESULTS

We now apply the theory of the spin Hall effect of Pt and the spin Peltier of YIG to a specific geometry. The differential equation for the potential H^*, Eq.(8), has general solution in the form of exponentials

$$H^*(x) = c_1 e^{-x/l_M} + c_2 e^{x/l_M} \tag{17}$$

and the specific solution only depends on the boundary conditions. As shown in Fig.1 we take the Pt layer of thickness t_1 and the YIG layer of thickness t_2. The connection between the two media is put at $x = 0$. As we deal with two different media, to simplify the notation we drop the M subscripts and use (1) for the Pt spin Hall injector and (2) for the YIG conductor with spin Peltier effect. The solutions are derived by setting the magnetization currents equal to zero at the external interfaces $j_M(-t_2) = 0$ and $j_M(t_1) = 0$ and by requiring the continuity of the potential at the interface $H_1^*(0) = H_2^*(0)$ (valid for an ideal interface). Moreover at the interface we impose the condition on the magnetization currents $j_1(0) = j_2(0) = j_0$. The magnetization current in the platinum injector (1) is

$$j_1(x) = j_{SH}\left(1 + \frac{\sinh(x/l_1)}{\sinh(t_1/l_1)}\right) + (j_0 - j_{SH})\frac{\sinh((x+t_1)/l_1)}{\sinh(t_1/l_1)}, \tag{18}$$

where l_1 is the diffusion length in Pt layer and is given by Eq.(9) with the values of the parameters for Pt. The potential is

$$H_1^*(x) = \frac{j_{SH}}{l_1/\tau_1}\frac{\cosh(x/l_1)}{\sinh(t_1/l_1)} + \frac{(j_0 - j_{SH})}{l_1/\tau_1}\frac{\cosh((x+t_1)/l_1)}{\sinh(t_1/l_1)}. \tag{19}$$

The magnetization current in the YIG conductor (2) is

$$j_2(x) = j_0\left(\cosh(x/l_2) - \coth(t_2/l_2)\sinh(x/l_2)\right) \tag{20}$$

where l_2 is the diffusion length for YIG in adiabatic conditions given by Eq.(16) with the values of the parameters for YIG. The potential is

$$H_2^*(x) = \frac{j_0}{l_2/\tau_2}\left(\sinh(x/l_2) - \coth(t_2/l_2)\cosh(x/l_2)\right). \tag{21}$$

By setting $H_1^*(0) = H_2^*(0)$ we obtain

$$j_0 = j_{SH} \frac{\cosh(t_1/l_1) - 1}{\cosh(t_1/l_1) + r\sinh(t_1/l_1)\coth(t_2/l_2)}, \tag{22}$$

where $r = (l_1/\tau_1)/(l_2/\tau_2)$. The temperature profile in YIG, from Eq.(15) is finally

$$T(x) - T(0) = \frac{1}{\varepsilon_2} \frac{\kappa_2}{\kappa + \kappa_2} \mu_0 \left(H_2^*(x) - H_2^*(0)\right) \tag{23}$$

where κ_2 is given by Eq.(12). Figures (2)-(3) show, respectively, the profile of the magnetization current and the effective field along the Pt/YIG bilayer for different value of the ratio t_1/l_1 as a function of the coordinate x normalized over the platinum diffusion length l_1. It is possible to clearly appreciate how the spin accumulation close to the boundaries generates, as a reaction, an effective field in order to let the current to go to zero at the interface.

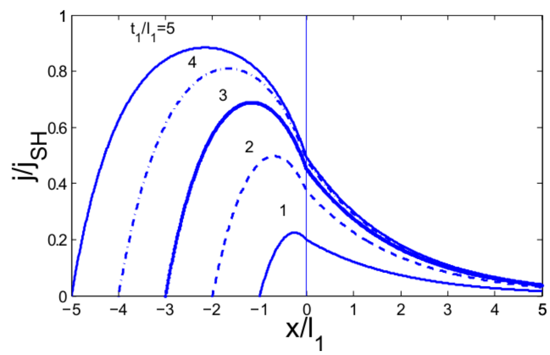

FIG. 2. Magnetization current profiles from Eqs. (20) and (18). The parameters are $r = 1$, $t_2/l_2 = 100$, $l_1/l_2 = 0.5$.

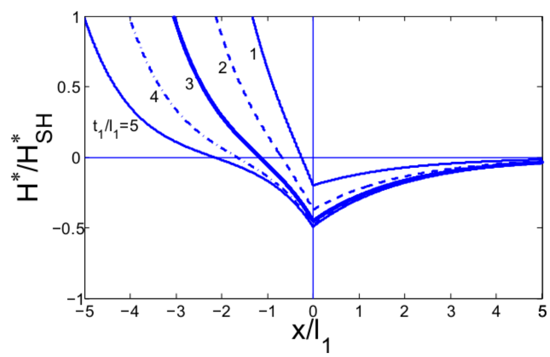

FIG. 3. Effective field profiles from Eqs. (21) and (19) with $H^*_{SH} = j_{SH}/(l_2/\tau_2)$. The parameters are $r = 1$, $t_2/l_2 = 100$, $l_1/l_2 = 0.5$.

From Eq.(22) it is possible to analyze the conditions for an efficient injection of magnetization currents from Pt into YIG. Let us observe that taking the limit $l_2 \ll t_2$, the

magnetization current of the YIG (Eq.(20)) becomes an exponential decay. From this consideration we may affirm that the injection into the YIG is effective only under the conditions $l_1 < t_1$ and $l_1/\tau_1 < l_2/\tau_2$. These conditions depend on the intrinsic properties of the media and on the thickness of the layers. Figure (4) shows the temperature profile from Eq.(23).

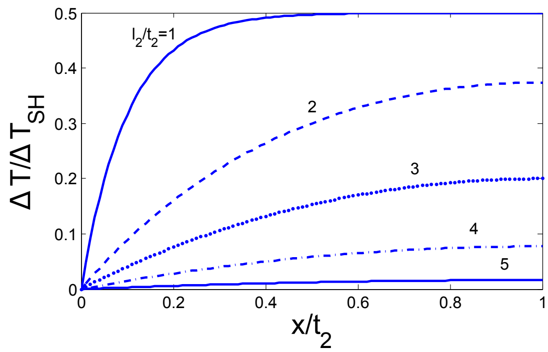

FIG. 4. Temperature profile from Eq.(23) with $\Delta T_{SH} = (1/\varepsilon_2)(\kappa_2/(\kappa + \kappa_2))\mu_0 H^*_{SH}$. The parameters are $r = 1$, $l_1/t_1 = 0.1$.

IV. CONCLUSION

Having derived the magnetization current profile injected into YIG and the temperature profile caused by the spin Peltier effect, we arrive at the following conclusions. The efficiency of the injection is based on three conditions. The thickness of each layer should be larger than its diffusion length: $t_{Pt} > l_{Pt}$ and $t_{YIG} > l_{YIG}$ in oder to permit to the magnetization current to develop in the conductor. Moreover the efficiency of the injection from Pt into YIG is regulated by the ratio of intrinsic parameters $r = (l_{Pt}/\tau_{Pt})/(l_{YIG}/\tau_{YIG})$. The magnetization current at the interface is large if the ratio r is small, i.e. the l_M/τ_M of the injector (Pt in our case) is much smaller than the conductor (YIG). The final conclusions on the spin Peltier effect can drawn by looking at the temperature profile obtained in adiabatic conditions of Fig.4. The profile is normalized to the temperature ΔT_{SH} which gives the typical scale of the effect

$$\Delta T_{SH} = \frac{1}{\varepsilon_M} \frac{\kappa_M}{\kappa + \kappa_M} \mu_0 H^*_{SH}. \tag{24}$$

By using approximate values from the literature, for YIG, we have $\varepsilon_M \simeq 10^{-2}$ TK^{-1} and $\kappa_M \simeq 10^{-2}$ W K^{-1}m^{-14}. The thermal conductivity of YIG is $\kappa = 6$ W K^{-1}m^{-1}. The

potential H^*_{SH} is related to the spin Hall current j_{SH} injected from Pt. Its expression is

$$H^*_{SH} = \frac{j_{SH}}{(l_{YIG}/\tau_{YIG})} \tag{25}$$

where j_{SH} is related to the electric current density in Pt along $\hat{y}$, j_{ey}, by $j_{SH} = -(\mu_B/e)\theta_{SH} j_{ey}$. We are then able to give an order of magnitude estimate of the temperature change. Using the values (from[4]) $l_{YIG}/\tau_{YIG} = 3$ ms^{-1}, $\theta_{SH} = -0.1$ we obtain $\Delta T_{SH}/j_e = 4 \cdot 10^{-13}$ K A^{-1}m^2.

In Ref.[2], for an electric current density of $3 \cdot 10^{10}$ A m^{-2} in Pt, the temperature difference measured by a thermocouple in YIG was $2.5 \cdot 10^{-4}$ K. This value is the result of the pure spin Peltier effect as the Joule heating of the electric current in Pt was already subtracted. Even if in the experiments the heat flux flows in two dimensions (x, z) rather than in a single dimension x as used here, we can still employ the present theory to obtain an order-of magnitude check. The parameter ΔT_{SH} results $1.2 \cdot 10^{-2}$ K which is of the correct order of magnitude. Furthermore by using $t_{Pt} = 5$ nm and $t_{YIG} = 0.2\,\mu$m, in Eqs.(21), (22) and (23), we find an adiabatic temperature change of $T(t_{YIG}) - T(0) \simeq 2.5 \cdot 10^{-4}$ K with $l_{YIG} = 0.4\,\mu$m. This value refines the upper limit of $1\,\mu$m which was found before.

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